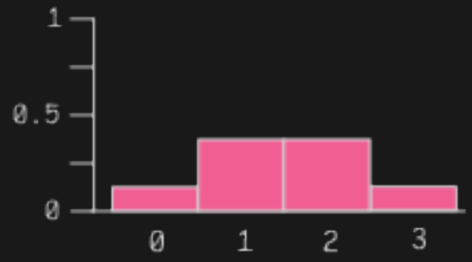
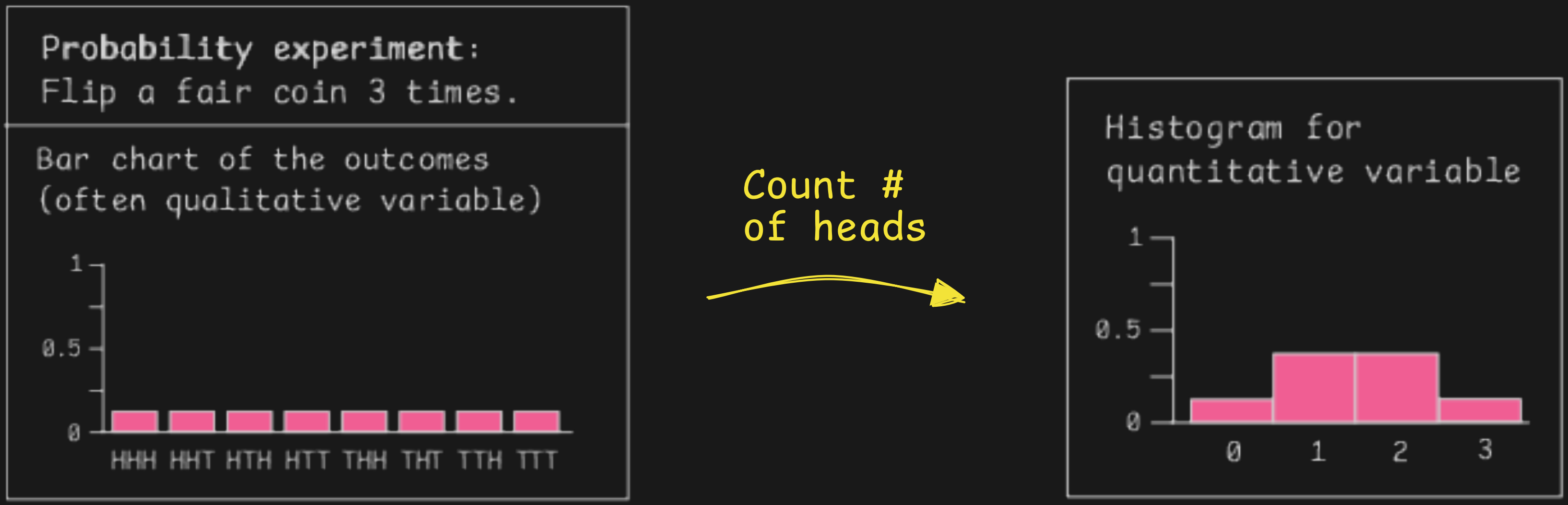
RANDOM VARIABLES

Probability experiment:
Flip a fair coin 3 times.

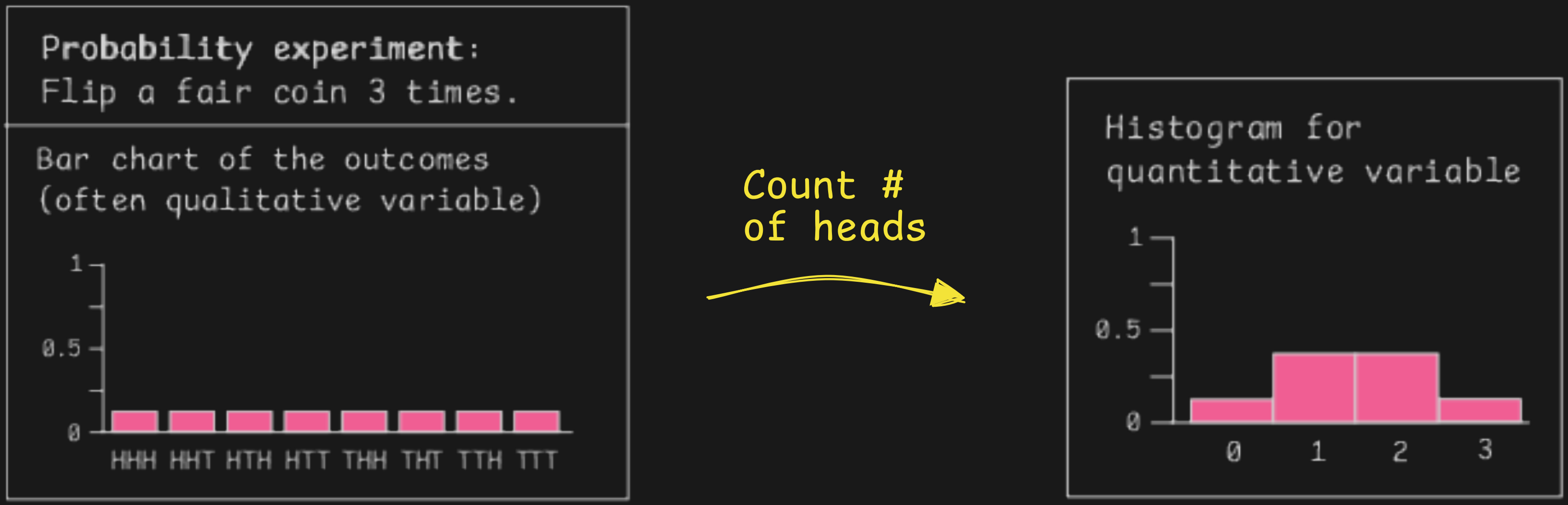
Bar chart of the outcomes
(often qualitative variable)

Histogram for
quantitative variable





Probability experiment:
Flip a fair coin 3 times.
Bar chart of the outcomes
(often qualitative variable)
1
0.5
0
HHH HHT HTH HTT THH THT TTH TTT
Count #
of heads
Histogram for
quantitative variable
1
0.5
0
0 1 2 3



Formally, a **random variable** converts qualitative/ quantative variable to quantitative variable.

But we think of a **random variable** as the resulting quantitative variable.

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$P(X = 3)$ = probability of 3 heads $= \frac{1}{8}$ HHH

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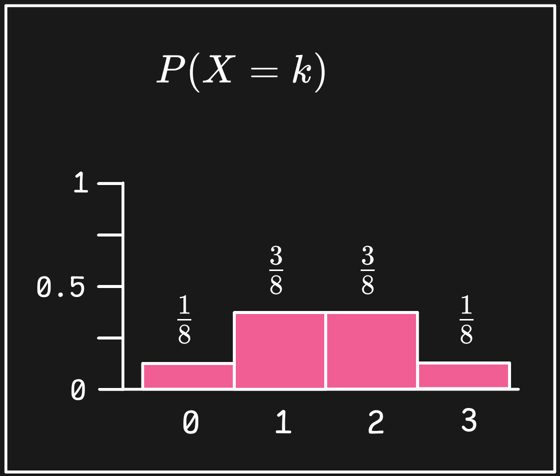

Probability distribution or probability mass function (PMF) of X

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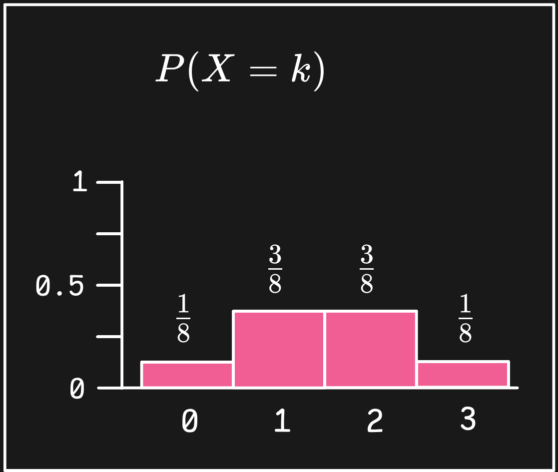

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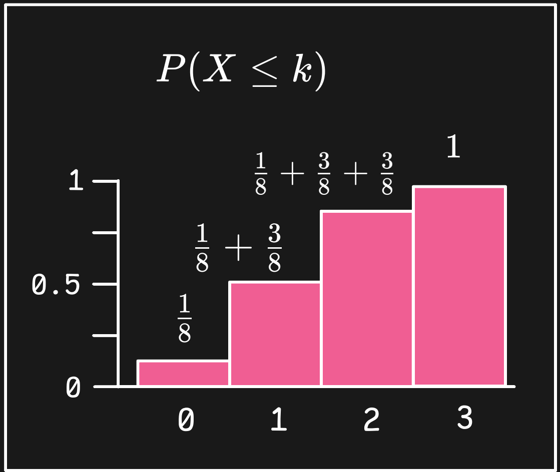

Cumulative distribution function (CDF) of X

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$$\Rightarrow P(X = 2) = \tbinom{10}{2}(0.25)^2(0.75)^8$$

General geometric RV $Y \sim \text{Geometric}(p)$:

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$$\Rightarrow P(Y = 4) = (5/6)^4 \cdot (1/6)$$

In-class exercise

Graph the PMF and CDF of the geometric random variable $Y \sim \text{Geometric}(1/2)$ where probability p of heads (aka success) is $1/2$.

Binomial and geometric RVs are discrete RVs.

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But there are also continuous RVs.

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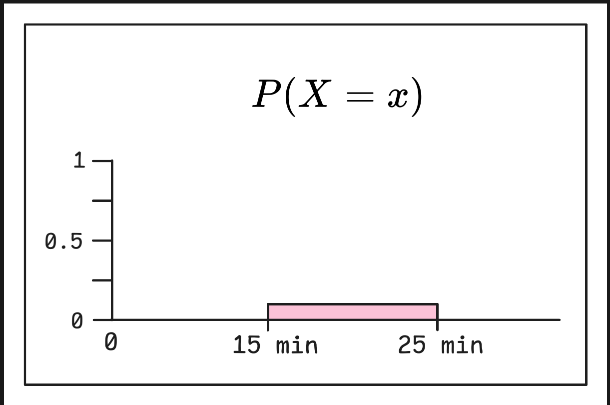
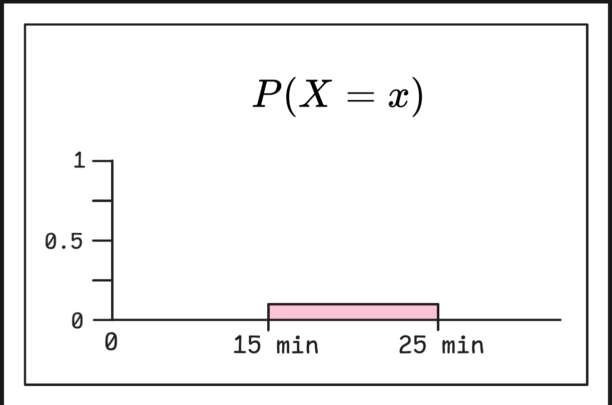


Probability density function (PDF) of X

Domino's Pizza is done at a time X that varies uniformly between 15 and 25 min.

PDF of X is constant with total area 1, so $f(x) = p(X = x) = 1/(25 - 15) = 0.1$ if $15 \leq x \leq 25$ and $f(x) = 0$ otherwise.



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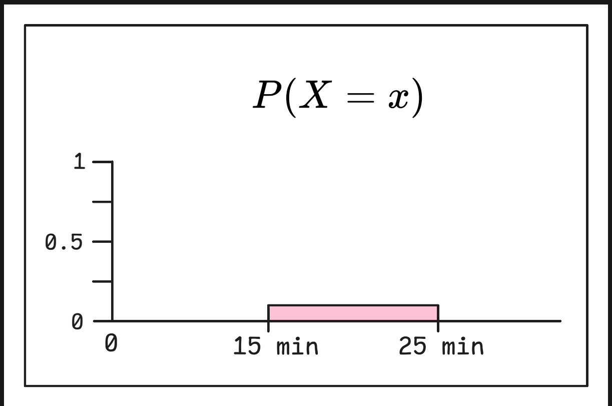

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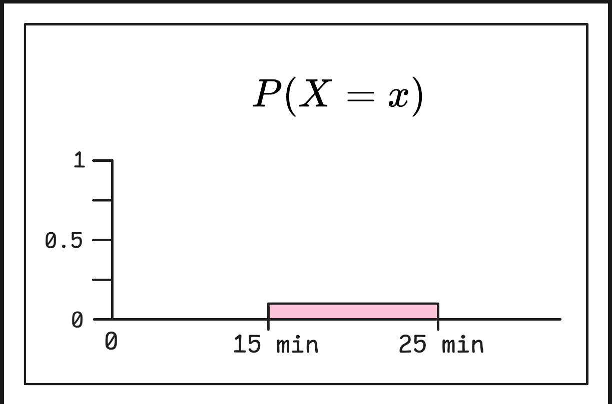

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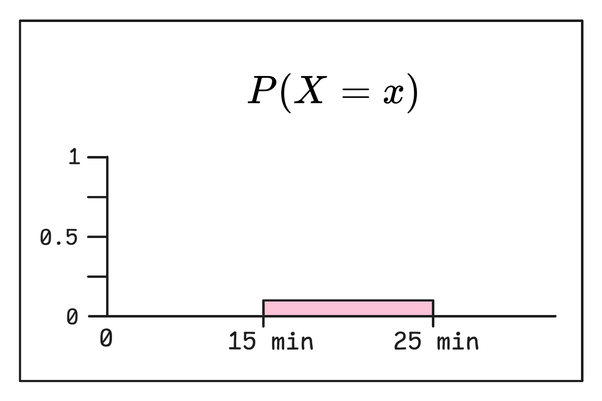

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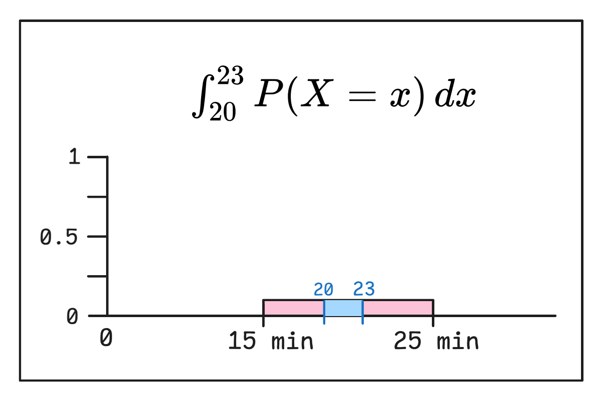

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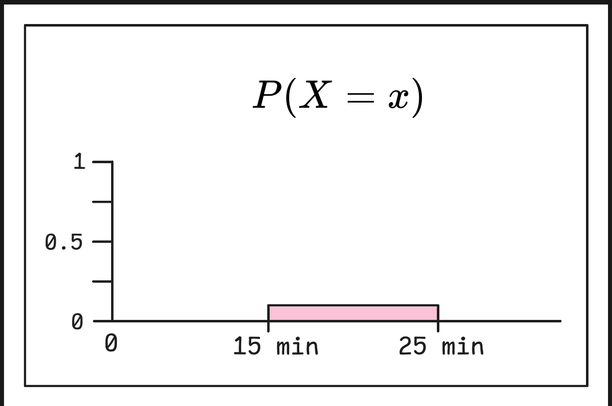

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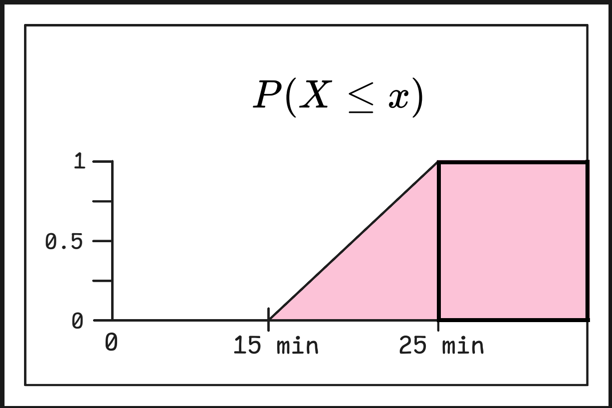

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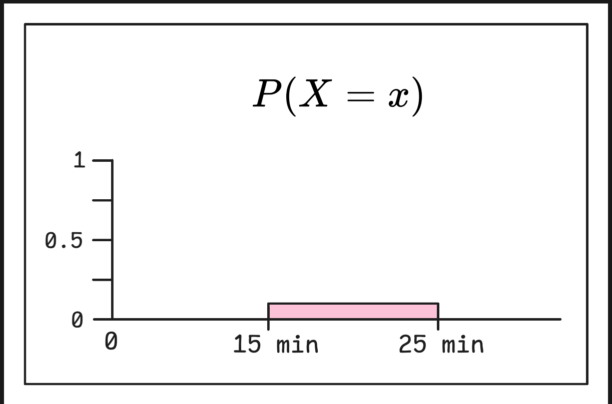

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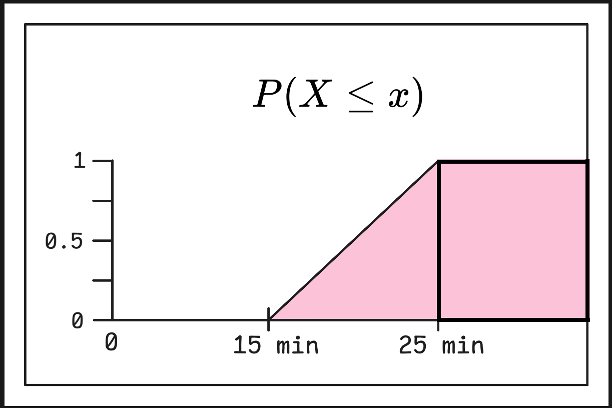

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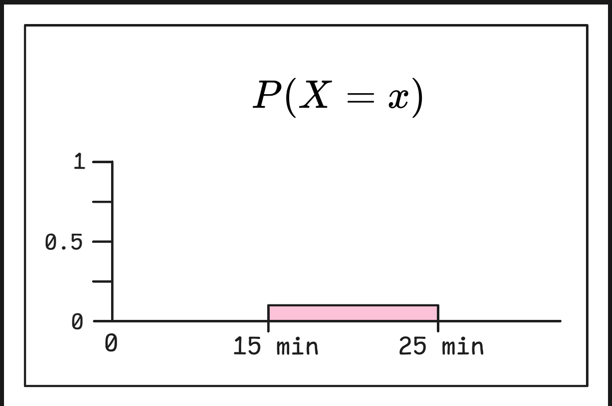

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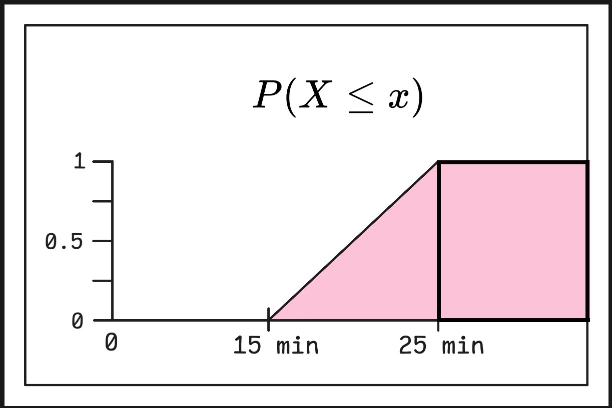

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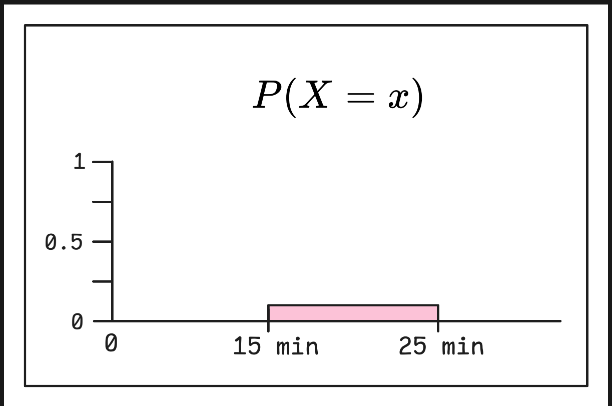

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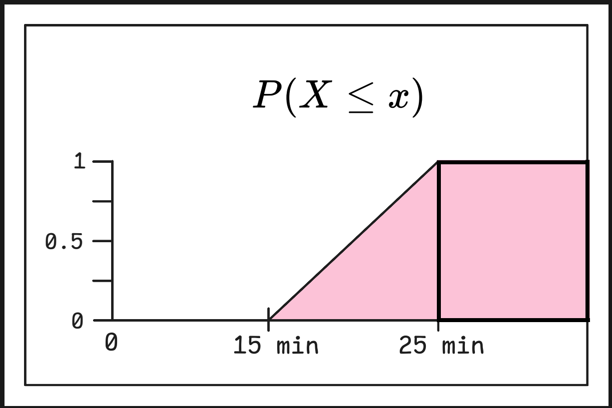

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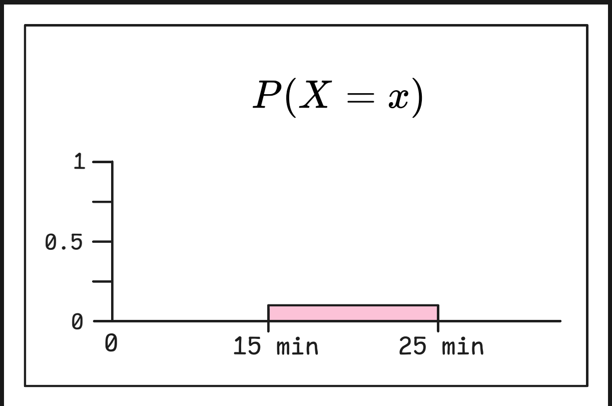

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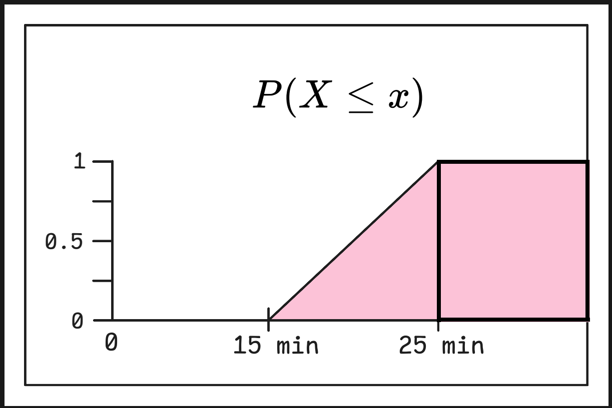

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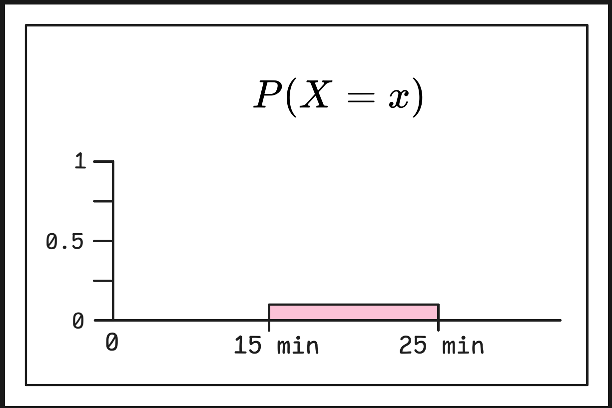

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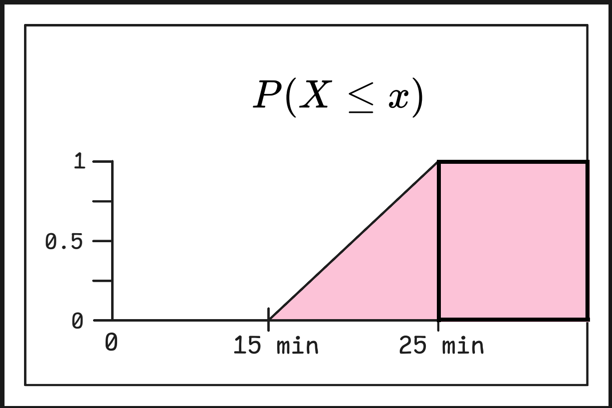

Cumulative distribution function (CDF) $F(x)$ of X

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PDF $f_X(x)$ of a continuous RV X is a non-negative function with total area 1.

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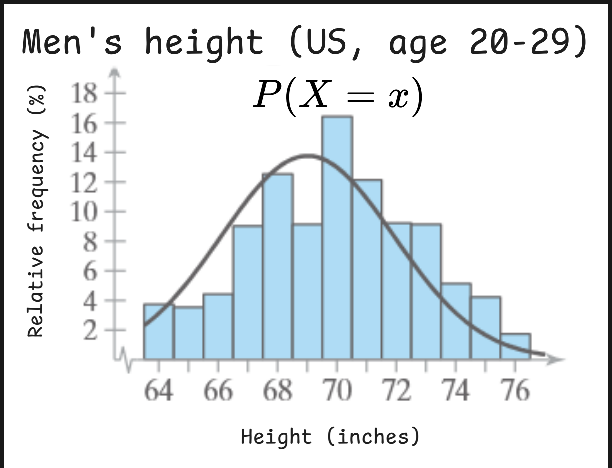
Uniform RV $X \sim U(a, b)$ between a and b has PDF $f_X(x) = \dfrac{1}{b-a}$ if $a \leq x \leq b$ and $f_X(x) = 0$ otherwise.

Normal RV $X \sim N(\mu, \sigma)$:

Let X be the heights of adult male. Then X is approximately normally distributed with $\mu = 70 \text{ in.}$ and $\sigma = 3 \text{ in.}$ Then

$$P(67 \leq X \leq 73) \approx \qquad .$$



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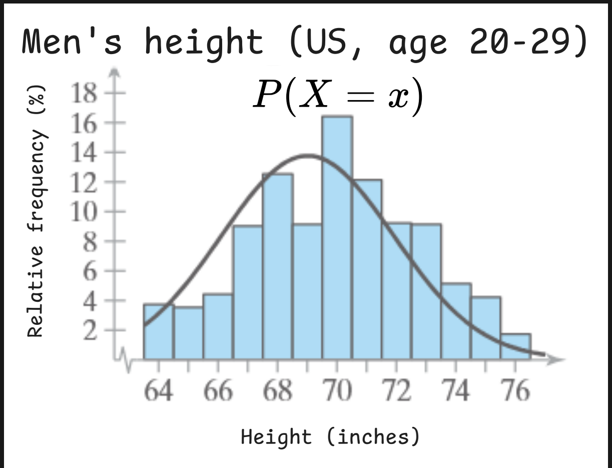

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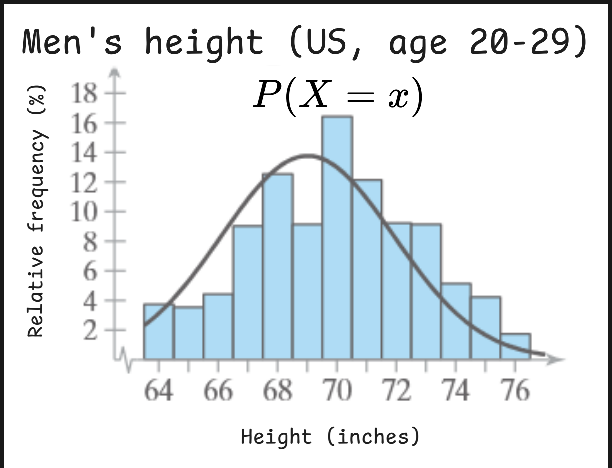

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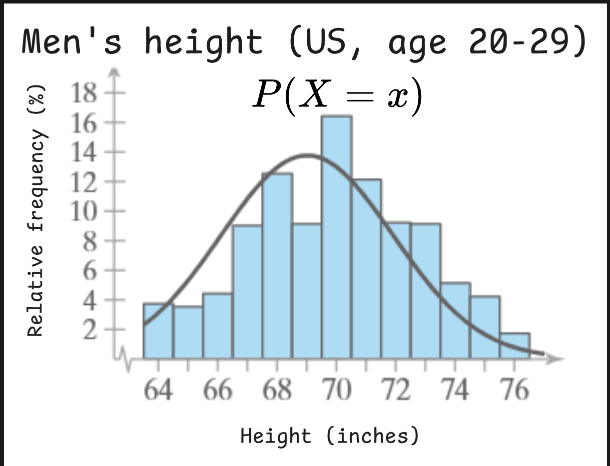

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(c) $\frac{1}{2} = F(x) = \sqrt{x} \Rightarrow x = \frac{1}{4}$.